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Asset Accumulation and Short Term Employment[1]

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Abstract

If access to credit is limited (especially when young or unemployed) but "bad" jobs are easy to come by, then job seekers might use short term employment in undesirable jobs as a way to finance consumption during subsequent unemployed search for a "good" job. In this paper we explore this idea in two ways. First, we document empirical patterns of short term employment and asset accumulation among job seekers. Second, we build a theoretical model of job search by risk averse, credit constrained agents. In this model we are able to demonstrate analytically that voluntary planned quits can occur as agents cycle between accumulating assets in short term employment and unemployed search for more desirable employment.

JEL Classifications: D83, D91, J64

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1 Introduction

Economists have long known that entrants to the labour market exhibit high turnover (for example, Topel and Ward, 1992, and Neal, 1999). A growing literature documents the propensity of workers who lose jobs to move to temporary employment, to have subsequent separations, and to move into part-time or otherwise unsatisfactory employment. For example, Farber (1999) shows that displaced workers commonly take up temporary jobs and "involuntary" part-time jobs, and Boheim and Taylor (2002) show that jobs that follow an unemployment spell have shorter average durations than other jobs. Farber further shows that the probability of temporary or part-time work falls with time since displacement, suggesting that these arrangements are part of a transitional process back to desirable employment.[1] How can apparently unsatisfactory employment aid the transition to more desirable jobs? Suppose access to credit is limited (especially when young or unemployed) but that "bad" jobs are easy to come by. In such an environment, job seekers might use short term employment in undesirable jobs as a way to finance consumption during periods of search for a "good" job. We explore this idea in two ways. The first half of the paper presents an empirical analysis of short term employment, consumption and saving after job loss. While a number of studies document the prevalence of short-term and temporary work (see above), we are not aware of any previous analysis that examines the empirical relationships between consumption, saving and short term

[1] Booth, Francesconi and Frank (2002) provide a recent analysis of the general incidence of temporary jobs in the U.K. These authors, and others, use "temporary job" to refer to fixed term contracts. We will use "temporary employment" and "short term employment" to refer more generally to employment spells of limited duration, including spells that end with a quit.

employment after job loss. In the second part of the paper we construct a theoretical model of job search by risk averse, credit constrained agents in an environment where "bad" jobs are readily available. In this model we demonstrate analytically that voluntary planned quits can occur as agents cycle between accumulating assets in short term employment and unemployed search for a more desirable employment. As the reader will recognise, our presentation is somewhat unusual in that we first present some empirical results and then a theory that is designed to rationalise these (and other) findings. At present the theory is not sufficiently general to allow us to estimate a credible structural model of job search, temporary employment and asset accumulation.

The empirical analysis, in Section 2 of the paper, is based on the 1995 Canadian Out of Employment Panel. This survey has an unusually broad coverage which allows us to document patterns of labour force transitions, consumption and saving after job loss. Beginning with labour force transitions, we show that quits into unemployment do occur. We also document (like Farber) the extensive use of temporary jobs after job loss. Turning to consumption and spending, we find that the expenditure level (measured as a fraction of pre-displacement expenditures) of workers in temporary, unsatisfactory employment is markedly different from those of workers who report being back in "good" jobs. The distribution of proportional expenditure changes among workers in temporary, unsatisfactory employment is actually quite similar to the distribution among unemployed job-searching individuals. However, temporary workers have higher incomes than searchers and thus, they have higher savings rates. This observation seems counter-intuitive: "permanent income" models suggest that those with temporarily low incomes

should have high average propensities to consume. On the other hand, it is consistent with the idea that temporary employment is used to finance subsequent search.

The idea that credit constrained job seekers use short term, "bad" jobs to finance further search is not readily incorporated into the standard search models that inform much thinking about unemployment. In particular, employment in such models is permanent (unless there is a subsequent shock) and job seekers are assumed to have risk neutral preferences and hence maximize the expected utility of the present discounted value of income less costs. This specification implies that the pattern of consumption is either indeterminate or completely insured. It also generates a stationary reservation wage strategy. A wage offer rejected today will not be acceptable in the future and any accepted job opportunity will always be preferred to further search (Mortensen, 1986). As a result, the transition rate from unemployment into employment does not vary with the duration of search or with differences in wealth. In addition, workers in such an environment only leave a job if they find a better opportunity (through on-the-job search) or if the employment relationship changes in some way so that the payoff to continued work becomes less productive than the return to search.

For risk averse job seekers without full insurance, much of this characterization does not apply. In this case, the optimal pattern of consumption becomes well defined. Individuals with higher assets consume more. In addition, the job acceptance decision also depends on the individual's asset levels although further restrictions on the degree of risk aversion are needed to characterize this relationship. Job seekers with decreasing absolute risk aversion become pickier as assets increase (the reservation wage declines

with asset holdings). As a result, wealthier individuals search longer and hence tend to match with higher paying jobs (Danforth, 1979). In addition, as risk averse job seekers consume from assets and become less picky, they may on occasion want to recall previously rejected offers (Hall, Lippman and McCall, 1979). However, as with risk neutral job search, employment is permanent: unless something changes for the worker, there are no job-to-unemployment transitions.

In contrast, in the model developed in the second part of the paper, credit constraints can give rise to employment-to-unemployment turnover. Lacking the ability to borrow or insure, risk averse individuals become willing to accept readily available, low wage "bad" jobs in order to accumulate assets which subsequently fund search for high wage "good" jobs. If the ensuing search for high wage employment proves unsuccessful and assets become depleted, these individuals then take up bad jobs again and repeat the cycle. Bad jobs act as a floor or safety net for those who are unsuccessful at job search and do not have the ability to borrow. As a result, temporary employment in bad jobs generates quits without shocks to productivity or in the job search process. Job seekers would prefer borrowing against future income to fund consumption during search rather than using low wage work to build assets. Credit constraints prevent such behavior and hence take on a central role in the determination of income distribution. Rather than limit human capital investment as often discussed in the literature, here they restrict the extent of search and thereby the flow of workers into desirable high wage jobs. Section 3 develops our model of short term employment and asset accumulation. The central result is that in such a model, cycles of search and temporary employment can arise

endogenously.

Section 3 also theoretically analyzes the effects of changes in turnover costs and wages. In our model turnover costs play a key role in determining asset accumulation as well as the durations of employment and job search. In particular, higher turnover costs lead to a longer employment spells in low wage jobs. While in these bad jobs, individuals facing high turnover costs accumulate a larger stock of assets but save a slower rate. When they do search, they are more cautious. They run down assets at a slower rate (by consuming less) and thereby search longer. Section 4 contains a concluding discussion.

2 Empirical Analysis

Our empirical analysis is based on the 1995 Canadian Out of Employment Panel (COEP). These surveys sample from the *flow* out of jobs and are intended to be representative of those who experienced a job separation in particular windows of time. The 1995 COEP had 3898 respondents in a first cohort (with separations between January and March 1995) and 3996 respondents in a second cohort (with separations between April and June 1995). Respondents completed two telephone interviews, conducted in approximately the 3rd and 5th quarter after job loss. Two features of the COEP data are particularly useful for our purposes. First, because they sample from the flow of job separations they contain unusually detailed information on these transitions. Second, the survey is unusual in that it contains data on expenditures after job loss, which allow us to relate patterns of consumption and savings to short term employment. Additional details about the COEP

surveys can be found in Browning and Crossley (2001a).

The COEP sampled almost all kinds of job separations[2] and the flow out of employment is very heterogeneous. Table 1 documents aspects of this heterogeneity. We initially deleted the 6% of respondents who reported continuing employment in a second job across the job loss. Of the remaining sample, respondents who reported being laid off due to business conditions are the largest group (see the last two columns of Table 1). But they constitute only 25% of the sample. There are also significant numbers who reported that they were in a temporary or seasonal job; that they quit, either to take another job or otherwise; that they were fired or dismissed; or that they left their job due to illness or injury.

For each of these groups Table 1 presents not just their importance in the sample, but also aspects of their post-job-loss experience. In particular, we report the median spell of initial jobless-ness (in weeks); the fraction of the group that searched for a new job after separation; and two measures of search intensity. Our measures of search intensity are usual hours per week spent searching and usual weekly job search related expenditures. In both cases we report the mean for those who report searching. We also report from each group the fraction that report a geographic move after job separation, and the fraction that received unemployment insurance benefit.

Table 1 contains a number of striking features. First, after excluding those that quit to take another job, other quits comprise a small, but not trivial, fraction of the sample (7%). Moreover, "other" quits (excluding those that quit to take another job) exhibit a median

[2]For the 1995 COEP Survey, retirement, maternity, labour dispute and return to school as reasons for separation were treated as out of the scope of our analysis.

initial spell of jobless-ness, a fraction searching, and measures of search intensity which are equal to or only slightly below the comparable numbers for respondents who were laid off because of business conditions. These "other" quits have a median initial spell of joblessness of 6 weeks (versus 7 weeks for business conditions layoffs) and 58% of them search for a new job after the separation (versus 72 % for business conditions layoffs). If they search, they spend on average 15 hours per week searching (identical to the business conditions layoffs) and spend on average 32 dollars per week on search related expenses (versus 34 dollars for business condition layoffs.) Thus the data contradict the notion that quits into unemployment do not occur. Quits do have lower incidence of unemployment insurance receipt than layoffs of any kind. In part this reflects the rules of the Canadian Unemployment Insurance system. Interestingly, these "other" quits (excluding those that quit to take another job) have the highest rate of geographic mobility of any group in the data.

A further noteworthy feature of Table 1 is the importance in the flow of job separations of workers leaving temporary jobs and expiring contracts. This group is almost as large as the group that best corresponds to traditional notion of job loss (those who were laid off due to business cycle conditions). This again emphasizes the importance of short-term employment, but in a slightly different way than the existing literature. In the model which we develop in the second half of the paper, accepting a job of fixed duration is equivalent to a planned quit into unemployment (because no information is revealed during employment.)[3]

[3]This would not be true of models in which information - such as the quality of the match - is revealed

The remainder of our analysis focuses on a restricted sample of "displaced workers" who were permanently displaced from a full-time job which they had held for at least one year. One reason to do this is to get a sample of new entrants to the search process (and so minimize initial conditions problems.) A second reason is that for all such individuals we get a measure of their household expenditures (consumption) when they are in a "good" job. We use this quantity to normalize expenditures in other states, as described below.

First, 1.5% of respondents under twenty or over sixty-five years of age were discarded. Involuntary separations (laid off or fired/dismissed) were selected which reduced the sample by a further 32%. A large fraction (62%) of the layoffs in the COEP data reported that, at the time of the layoff, they had an expectation of recall to the firm that was laying them off. Many also reported that they had a specific recall date. In order to focus on displaced workers, respondents who reported such an *ex ante* expectation of recall (with or without a recall date) were dropped.[4] The final sample restriction was to select respondents who were displaced from stable, full-time employment. About half of the remaining sample had tenure of less than 52 weeks in the lost job, and these were dropped. So were the approximately 15% of the sample that had been displaced from a part-time job. This left a sample of 790 displaced workers. About 16% of these withdrew from the labour force after job loss. These are not analyzed in what follows.

Table 2 documents short run employment outcomes of displaced workers in the 1995

during a spell of employment.

[4]Throughout the paper we use the terms "permanent layoff" and "displacement" interchangeably. Some of the literature uses a tighter definition of "displaced workers," limiting the term to those laid off as part of a plant closing or other large event.

COEP. Six to ten months after the loss of a good job, 33.3% are still in the spell of unemployment that began with that job loss, while 37.6% are in the first spell of employment subsequent to that job loss. The remaining 29.1% of workers are either in a subsequent unemployment spell or a subsequent employment spell and thus have had some short-term employment.

Of course, some of those in their first spell of employment may also be in a temporary job. The data allow us to look at this in two ways. First, of those in the first spell of employment subsequent to a job loss, 29.3% do not expect to be in that job for a full year. Second, a substantially overlapping 24.9% consider this job to be not as good as the job they lost. Thus, in total, more than a third of workers who lose a permanent (long tenure) job take some short-term (and unsatisfactory) employment in the first three quarters after displacement. This is work that they either quickly leave or do not expect to remain in.

We now turn to the consumption and saving patterns among these job losers. Figure 1 displays distributions of proportional changes in household expenditures (consumption) from before the job loss to the first interview. Analyzing this transformation (rather than expenditure levels) removes heterogeneity across households in pre-displacement expenditure levels. Because we take these workers to have been displaced from "good" jobs, we are effectively normalizing by the level of expenditures in the good job. The sample is split into four groups: those in their first spell of unemployment after a displacement (UE-1), those in a subsequent spell of unemployment (UE-2), and two groups of employed who consider their job worse (E-1) or at least as good (E-2) as the previous job. For each

group, a "box and whiskers" plot is used to summarize the entire distribution . The box in each case represents the inter-quartile range (from the 25th to the 75th percentile) and the "whiskers" above and below the box indicate the 10th and 90th percentile of the distribution. The median is represented by the horizontal line across the box (in some cases the median is the same as the 25th or 75th percentile in which case there is no line). Extreme observations are represented by dots above and below the box and whiskers (see for example, McGill, Tukey and Larsen, 1978). The most striking feature of Figure 1 is that those in jobs that they report are "worse" than the job they lost exhibit a distribution of proportional expenditure losses which is distinctly different from those who rate their jobs the same or better. Moreover, those in "worse" jobs have proportional changes similar to those who are unemployed. Table 3 reports pair-wise t-tests of equality of means and pair-wise Kruskal-Wallis (rank) tests of equality between the distributions presented in Figure 1. These indicate that the statistical significance of the patterns seen in the figure. That groups UE-1, UE-2 and E-1 have the same distribution cannot be rejected at standard significance levels, but group E-2 has significantly higher values.

As well as consumption we also consider income. Figure 2 suggests that the changes in consumption are not simply a reflection of changes in income. While those in bad jobs have larger income losses than those in good jobs, they have a smaller median income loss than the unemployed. Table 4 confirms (with t-tests and rank tests) that the distribution of income changes among those in "worse" jobs is statistically different both from the distribution of expenditure changes among the other employed groups (in jobs rated the same or better) and from those in their first spell of unemployment.

Figure 3 and Table 5 report on the savings rates of individuals in the four groups defined above.[5] Here we see a striking and statistically significant difference between those in "worse" jobs and the unemployed: a higher saving rate. In fact, those in "worse" jobs seem to be saving at the same rate as those in good jobs, despite the fact that the former are in straightened circumstances - with considerable earnings losses.

To summarize, the COEP data add to a body of evidence pointing to the importance of temporary jobs and short-term employment during periods of labour market entry and adjustment. They also bring to light a number of empirical patterns which have received little (if any) attention to date. In particular, the data contain evidence of quits into unemployed search. They also exhibit a striking pattern of consumption and savings across labour market states. Displaced workers in subsequent short term employment have low consumption and high savings.

Individually, these observations are compatible with a number of economic environments. For instance, models in which jobs are experience goods, as emphasized by Topel and Ward (1992), can explain rapid turnover on labour market entry or after a job displacement. On the other hand, these empirical patterns do not collectively match up with a number of standard labour market specifications. But they do fit the picture of

[5]The expenditure information in the COEP is collected by a series of recall questions. An analysis of the responses, reported in Browning and Crossley (2002), determined that these questions suffered from an under-reporting of expenditures - relative to income - and that this under-reporting was largely independent of the level of expenditure. A corresponding adjustment is made in the calculation of the savings rates displayed in Figure 3. Note, however, that this proportional adjustment makes no difference to the *pattern* of savings rates across the four groups.

credit constrained job seekers using short-term employment to finance consumption during subsequent search. In the second half of the paper we present a model which captures this idea more rigorously.

3 A Model of Short Term Employment and Asset Accumulation

Time is continuous and infinite. Agents make consumption and employment decisions so as to maximize the expected value of discounted lifetime utility. Let $u(\cdot)$ represent the worker's risk averse preferences in each period. $u(\cdot)$ is continuous, differentiable and bounded above with the standard Inada properties so that $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. At any time an unemployed agent can accept a low paying job at a wage $w_L > 0$. Such jobs are always available. Unemployed agents who forgo the low wage sector (and only these) can search for higher paying employment with the associated wage $w_H > w_L$. As discussed in the conclusions, the assumption that there is no search on the 'bad' job is largely innocuous but convenient. Flow payments during unemployed search from sources such as unemployment insurance are normalized to zero. When looking for a job, a job seeker receives a high wage offer with probability α. The key feature here is that low wage jobs are easier to find than high wage jobs. We assume that once found, a high wage job lasts forever; this is an unattractive assumption but, as discussed in the conclusions, it makes the model tractable enough for us to be able to derive analytical results.

Suppose that at time t a worker has assets $A_t \geq 0$. A worker entering the market for the first time does so with assets $A_0 \geq 0$. Job seekers earn interest on these assets at rate r which also equals the individual's rate of time preference. Although accepting a low wage job is costless, it is not costless to embark on search. Entrants into the market who decide to search immediately as well as low wage workers who quit employment in order to search pay a turnover cost $K > 0$. Throughout this analysis we refer to the end of a low wage job as a quit. It is important to note, however, that in this model no information is revealed during low wage employment, and the timing of a quit is fully anticipated at the start of a job. Thus in this model taking and then quitting an ongoing job is equivalent to accepting a fixed term contract of the same duration.

Turnover costs have a number of possible interpretations. For transitions from low wage employment, the most immediate corresponds to an up-front search cost as well as an exit cost borne by the worker. In the context presented here, a search or exit cost of this sort is equivalent to an entry fee in the low wage sector properly adjusted in present value terms.[6] An alternative view is that these costs represent (round trip) transportation costs between spatially distinct sectors, the low wage - full employment and the high wage with search unemployment sectors. This perspective highlights the similarities of this model with that of Harris and Todaro (1970). Here, however, the economy explicitly accounts for the dynamic flows between sectors. Of course, the act of

[6] As discussed below, a worker with sufficiently high assets will initially search. In this case, there is no exit from (or entry into) the low wage sector to generate the cost K. As the focus here is on flows between job search and low wage employment, this fault in the interpretation of K is of minor concern.

changing sectors need not be explicitly spatial but nonetheless involve a transportation cost.

Given this setting, consider first the "partial" problem of a job seeker (an individual in the job search process) with assets A_D which are at this point given. $A_D \geq 0$ may differ from initial assets A_0 if the worker initially participates in the low wage sector. While looking for high wage employment, the worker's problem is to choose the maximum duration of search T (given a job is not found), a consumption path, c_t for $t \in [0, T]$, and the asset level desired at the end of the search period, $A_T \geq 0$. These decisions are made bearing in mind the opportunities of low wage employment, represented here by the value of low wage employment, $V(A)$.

As high paid jobs last forever, a worker with assets A_t in a high wage job will optimally consume $rA_t + w_H$ indefinitely. The corresponding value of high wage employment is therefore $u(rA_t + w_H)/r$. As a result, the searching worker's problem can be written as[7]

$$W(A_D) = \max_{c_t, T, A_T} \int_0^T u(c_t)e^{-(r+\alpha)t}dt + \frac{\alpha}{r}\int_0^T u(rA_s + w_H)e^{-(r+\alpha)s}ds + e^{-(r+\alpha)T}V(A_T)$$

[7]The objective function can be viewed as the limit of a related discrete time problem. Given a time period of length $dt > 0$, the worker receives the expected payoff

$$W(A) = u(c_0)dt + \sum_{t=1}^{T}\left(\frac{1-\alpha dt}{1+rdt}\right)^t u(c_t)dt + \quad \frac{\alpha dt}{1-\alpha dt} \quad \sum_{t=1}^{T}\left(\frac{1-\alpha dt}{1+rdt}\right)^t u(rA_t + w_H)/r$$
$$+\frac{1}{1-\alpha dt}\left(\frac{1-\alpha dt}{1+rdt}\right)^T V(A_T)$$

Letting $dt \to 0$ yields the continuous time objective function.

subject to

$$A_D - \int_0^T c_t e^{-rt} dt - e^{-rT} A_T \geq 0 \qquad (Budget\ Constraint)$$

and

$$e^{-rT} A_T \geq 0 \qquad (Credit\ Constraint)$$

where

$$A_s = e^{rs} \left[A_D - \int_0^s c_v e^{-rv} dv \right] \geq 0 \qquad s \in [0, T]$$

Characterizing the value of working in low wage jobs, $V(A)$, completes the specification of the worker's decision problem. Workers in low wage employment have the option of turning at some time to high wage job search at a cost K or remaining indefinitely in low wage employment with no intention of further search. A worker in low wage employment with initial assets A_0 chooses a duration of employment D, a consumption pattern, ς_τ for $\tau \in [0, D]$, and a terminal level of assets $A_D \geq 0$. If the worker turns to high wage job search ($D < \infty$), terminal assets fund the turnover cost K and the subsequent consumption while searching.

The worker's problem is thus expressed by[8]:

$$V(A_0) = \max_{\varsigma_\tau, D, A_D} \int_0^D u(\varsigma_\tau) e^{-r\tau} d\tau + e^{-rD} W(A_D)$$

[8]There are also implicit nonnegativity constraints on the choice variables. As discusssed below, these are potentially binding only for the duration of low wage employment ($D \geq 0$).

subject to

$$A_0 + \int_0^D (w_L - \varsigma_\tau) e^{-r\tau} d\tau - e^{-rD} A_D - e^{-rD} K = 0 \qquad (Budget\ Constraint)$$

and

$$e^{-rD}(A_D - K) \geq 0 \qquad (Credit\ Constraint)$$

Substituting in for $W(A_D)$ as well as for A_D in the budget constraint yields the following problem

$$V(A_0) = \max_{\varsigma_\tau, c_t, D, T, A_T} \int_0^D u(\varsigma_\tau) e^{-r\tau} d\tau + e^{-rD} \left\{ \int_0^T u(c_t) e^{-(r+\alpha)t} dt + \frac{\alpha}{r} \int_0^T u(rA_s + w_H) e^{-(r+\alpha)s} ds + e^{-(r+\alpha)T} V(A_T) \right\}$$

subject to

$$A_0 + \int_0^D (w_L - \varsigma_\tau) e^{-r\tau} d\tau - e^{-rD} \int_0^T c_t e^{-rt} dt - e^{-rD} K - e^{-r(D+T)} A_T \geq 0$$

and

$$e^{-r(D+T)} A_T \geq 0$$

where now

$$A_s = e^{rs} \left\{ e^{rD} A_0 + e^{rD} \int_0^D (w_L - \varsigma_\upsilon) e^{-r\upsilon} d\upsilon - \int_0^s c_v e^{-rv} dv - K \right\} \geq 0 \quad s \in [0, T]$$

3.1 Consumption Behavior

Associate Lagrange multipliers μ_1 and μ_2 with the budget and credit constraints respectively. The first order conditions for consumption while working in a low wage job (ς_τ) are[9]

$$e^{-r\tau}\left\{u'(\varsigma_\tau)-\alpha\int_0^T u'(rA_s+w_H)e^{-\alpha s}ds-\mu_1\right\}=0 \quad \tau\in[0,D]$$

These equations imply that (in the absence of on-the-job search) the individual will choose constant consumption over this period.[10] Denoting $\bar{\varsigma}=\varsigma_\tau$ for all $\tau\in[0,D]$ and rearranging terms, the first order conditions reduce to a single equation

$$u'(\bar{\varsigma})-\alpha\int_0^T u'(rA_s+w_H)e^{-\alpha s}ds-\mu_1=0 \tag{1}$$

[9]The asset equation for A_s implies that

$$\partial A_s/\partial\varsigma_\tau=-e^{r(D+s-\tau)} \qquad s\in[0,T] \qquad \tau\in[0,D]$$

$$\partial A_s/\partial c_t=\begin{cases}-e^{r(s-t)} & s\geq t\\ 0 & s<t\end{cases} \qquad 0\leq s,t\leq T$$

$$\partial A_s/\partial D=e^{rs}\left[re^{rD}A_0+re^{rD}\int_0^D(w_L-\varsigma_v)e^{-rv}dv+w_L-\varsigma_D\right]$$

$$\partial A_s/\partial T=\partial A_s/\partial A_T=0$$

[10]In the adopted notation here, the indices t, τ and s do not necessarily correspond to chronological time. For consumption during low wage employment, τ does at first match real time but for consumption while searching, c_t, this index differs from the date by D. Of course, if cycles of work and employment occur, the index further differs by a multiple of $T+D$.

For consumption during high wage job search (c_t), the first order conditions are

$$e^{-r(D+t)}\left\{u'(c_t)e^{-\alpha t}-\alpha\int_t^T u'(rA_s+w_H)e^{-\alpha s}ds-\mu_1\right\}=0 \quad t\in[0,T] \tag{2}$$

From equation (1) and equation (2) for $t=0$, consumption at the beginning of search equals consumption during low wage employment (if low wage employment is taken on) which in turn is less than or equal to the low wage plus initial asset income: $c_0=\bar{\varsigma}\leq rA_0+w_L$. It can also be established from (2) that consumption during search c_t is strictly positive for all t and decreasing over time. As search proceeds, consumption falls until either a high wage job is found or search terminates with corresponding consumption c_T. If the worker chooses to cycle back and forth between low wage employment and high wage job search, this consumption pattern implies that a jump in consumption occurs when low wage temporary employment begins anew but not when the worker quits low wage employment.

The jump at the end of search can be intuitively understood by considering the marginal value of using assets to finance an added unit of consumption today. When searching, the opportunity cost of consuming a little more today is having to give up search sooner. Toward the end of search, this means not being able to finance the search tomorrow. On the other hand, when working at a low wage, the opportunity cost of a little more consumption is getting back to future search a bit later. The discounting of postponed search creates a wedge that generates the jump. At the other extreme of quitting low wage employment, there is no such delay. The marginal value and therefore

the level of consumption in the two states are equal.

Consumption during search can be characterized further. Provided that search occurs, that is $T > 0$, equation (2) generates the differential equation

$$\dot{c}_t = \frac{\alpha \left[u'(c_t) - u'(rA_t + w_H) \right]}{u''(c_t)} \tag{3}$$

while the asset equation gives a second differential equation in assets and consumption

$$\dot{A}_t = rA_t - c_t \tag{4}$$

Figure 4 illustrates the associated phase diagram along with the equations for $\dot{A}_t = 0$ and $\dot{c}_t = 0$. Since these stationary lines are parallel at a distance of w_H from each other, there are no stationary points in this system. Moreover, it is straightforward to establish that the optimal solution lies between these two lines:

$$rA_t < c_t < rA_t + w_H.$$

As a result, for any terminal point (c_T, A_T), there is a unique path and any stable path has decreasing assets and consumption over time.

3.2 Search and Employment Duration

As $\varsigma_\tau = \bar{\varsigma}$ for all τ, the first order condition for the duration of low wage employment (D)

$$e^{-rD}\left\{u(\varsigma_D) - r\int_0^T u(c_t)e^{-(r+\alpha)t}dt - \alpha\int_0^T u(rA_s + w_H)e^{-(r+\alpha)s}ds\right.$$

$$-re^{-(r+\alpha)T}V(A_T) + \alpha\int_0^T u'(rA_s + w_H)(\partial A_s/\partial D)e^{-(r+\alpha)s}ds$$

$$\left.+\,\mu_1\left[w_L - \varsigma_D + r\int_0^T c_t e^{-rt}dt + rK + re^{-rT}A_T\right] - \mu_2 re^{-rT}A_T\right\} = 0$$

simplifies to

$$u(\bar{\varsigma}) - rV(A_0) + (rA_0 + w_L - \bar{\varsigma})\alpha\int_t^T u'(rA_s + w_H)e^{-\alpha s}ds \qquad (5)$$

$$+\mu_1(rA_0 + w_L - \bar{\varsigma}) \;=\; 0$$

The remaining first order conditions for duration of high wage search (T) and asset holdings at this time (A_T) are given by:

$$e^{-r(D+T)}\left\{u(c_T)e^{-\alpha T} + \frac{\alpha}{r}u(rA_T + w_H)e^{-\alpha T} - (r+\alpha)e^{-\alpha T}V(A_T)\right. \qquad (6)$$

$$\left.-\mu_1(c_T - rA_T) - \mu_2 rA_T\right\} \;=\; 0$$

$$e^{-r(D+T)}\left\{e^{-\alpha T}V'(A_T) - \mu_1 + \mu_2\right\} = 0 \qquad (7)$$

while the Kuhn-Tucker conditions for the two constraints are

$$\mu_1 \left\{ A_0 + \int_0^D (w_L - \varsigma_\tau) e^{-r\tau} d\tau - e^{-rD} \int_0^T c_t e^{-rt} dt - e^{-rD} K - e^{-r(D+T)} A_T \right\} = 0 \qquad (8)$$

$$\mu_2 e^{-r(D+T)} A_T = 0 \qquad (9)$$

The primary interest here are consumer choices of $\bar{\varsigma}, c_t$ for $t \in [0,T], D, T$, and A_T in which workers move back and forth between low wage employment and job search until finding a permanent high wage job. Of course, such a choice along with multipliers (μ_1, μ_2) must solve equations (1)-(9) for assets $A_0 = A_T$. However, not all solutions to these necessary (but not sufficient) conditions are optimal choices. More specifically, note that $D = \infty$ along with $\varsigma_t = \bar{\varsigma} = rA_0 + w_L$ is always a solution for an arbitrary level of initial assets $A_0 \geq 0$. In this case, c_t, T, A_T are undetermined as search does not occur. Therefore to establish existence of the type of solution of interest, we find conditions which rule out this solution and other alternatives.

The following claim establishes sufficient conditions on wages and turnover costs under which the $D = \infty$ solution to the first order conditions is suboptimal. Wages in the good jobs must be sufficiently high to make search attractive while the turnover costs must be sufficiently small so that the worker is willing to participate in high wage job search.

Claim 1 *If*

$$\frac{\alpha}{r} \left(u(rA_0 + w_H) - u(rA_0 + w_L) \right) - u'(rA_0 + w_L) w_L > 0,$$

then a worker will at some point switch to high wage job search ($D < \infty$) *given sufficiently small but strictly positive* K*.*

Proof: See Appendix

On the other hand, there may be no transition from high wage search into low wage employment. If search is very attractive, it may be optimal to search indefinitely, $T = \infty$. In this case, the job seeker runs down assets (recall that $c_t > rA_t$) so that consumption becomes arbitrarily small as time proceeds.[11] This action is ruled out when wages in good jobs are not "too attractive."

Claim 2 *If* $\frac{\alpha}{r+\alpha}u(w_H) < u(w_L)$*, then* $T < \infty$*.*

Proof: See Appendix

While Claim 1 establishes conditions for not remaining permanently in low wage employment with assets A_0, it does not demonstrate that a worker with these assets will necessarily take on and then later quit a low wage job. Given A_0, a worker may forgo low wage employment altogether and search immediately. Indeed, for some initial asset levels, low wage employment (accompanied by asset accumulation) is undesirable. Upon entering the market, it may be optimal to set $D = 0$.[12] Of course, following a period of

[11] Letting the $u(0)$ normalization (currently equal to zero) decrease will lower the attractiveness of search when assets are low thereby easing the conditions under which cycles exist.

[12] For assets less than K this is not feasible since the subsequent search does not occur: $T = 0$. For A_0 greater than some critical value this will indeed be a solution. More specifically, if workers begin with different endowments, A_0, those with high levels of initial assets will immediately search for high wages and only take up low wage jobs when high wage search is unsuccessful. On the other hand, workers with low endowments will accumulate assets before search ($D > 0$).

high wage job search, setting $D = 0$ is wasteful. A worker who discontinues high wage search to move into low wage employment with assets A_T will not then set $D = 0$ when solving $V(A_T)$. Acting in this way involves paying an avoidable turnover cost K.[13]

To assess what happens after pursuing of a good job, suppose a worker concludes search with assets A_T. In general, given A_T it may be optimal to set $D = \infty$. If, however, the condition in Claim 1 holds at A_T, then the worker will take on low wage employment for a only finite, strictly positive duration after which the worker will search yet again. This pattern suggests that temporary low wage employment cycles could emerge if the condition in Claim 1 holds over a range of assets. Claim 3 goes even further. It shows that if the condition holds more generally, then at the end of unsuccessful search, an individual will have exhausted the assets in which case the credit constraint binds.

Claim 3 *If*

$$\frac{\alpha}{r}\left(u(rA + w_H) - u(rA + w_L)\right) - u'(rA + w_L)w_L > 0 \quad \forall\ A \in [0, A_0]$$

then $A_T = 0$

Proof: See Appendix

When the conditions in Claims 2 and 3 simultaneously hold, the (repeated) pattern of low wage employment followed by high wage job search emerges. Moreover, since these two conditions are not exclusive (examples are easy to find), cycles can emerge.

[13] Likewise $T = 0$ is not part of an optimal plan as this strategy also involves paying a transition fee without any possible payoff from search.

Regardless of initial assets, workers will (with some probability) search until assets are used up. At this point, Claim 1 establishes that they will not take a low wage job permanently. Claim 2 establishes they will not search with zero assets. Instead they will take on low wage employment for a finite period after which they search. If unsuccessful, this search will terminate with zero assets at which point the process begins anew.[14] In other words, the solution to $V(0)$ is such that workers cycle indefinitely between low wage employment and high wage job search until they ultimately find a high wage job, thereby establishing the central result of this section.

This cyclical pattern of employment can be illustrated from the value functions. As depicted in Figure 5, the value of high wage search, $W(A)$, is strictly increasing and concave in assets A whereas the value of low wage employment, $V(A)$, is everywhere increasing but linear over the range of assets involving savings. (Derivations are shown in the proof of Claim 3 in the Appendix.) In Figure 5, starting from assets $A = 0$, a cycle begins with the accumulation to A_D assets and the corresponding progression up along $V(A)$. Upon reaching A_D, the individual becomes indifferent between low wage employment and moving to search: $V(A_D) = W(A_D - K)$. Here, a smooth pasting-type condition holds in which the value functions have the same slope, $V'(A_D) = W'(A_D - K)$.

As search proceeds assets decline from this point. The individual moves down along $W(A)$ until assets are exhausted at which point the individual switches: $V(0) = W(0)$. With a binding credit constraint, the marginal return to an additional asset is strictly greater under search than in low wage employment but the fixed cost K deters the

[14] For completeness, erratic patterns in which the choices of D and T vary can also be dismissed.

individual from working a very short period. Instead the process repeats itself until a high wage job is ultimately found.

When the credit constraint does not bind ($A_T > 0$), the solution is less involved. After any period of job search, the worker abandons the high wage market preferring (permanent) low wage employment. This switch occurs at a point of indifference which smoothly links $W(A)$ and $V(A)$ through a tangency condition. Specifically, if $A_T > 0$ after an initial period of search, then $W(A_T) = V(A_T)$ and $W'(A_T) = V'(A_T)$ which further implies that $D = \infty$. $A_T = 0$ is a necessary element of cyclical job quits.

3.3 Turnover Costs

How does consumption and the duration of job search respond to a change in turnover costs? Do job seekers begin search with higher assets? What are the consequences for unemployment? Since asset accumulation and job quits occur only in the case where workers who terminate job search do so when the credit constraint binds ($A_T = 0$), when considering these effects it is sufficient to concentrate on the case in which initial and terminal assets are zero.

As shown in the Appendix, in this cycle consumption during low wage employment increases with turnover costs implying that the initial level of consumption during search also rises:

$$\partial c_0/\partial K = \partial \bar{\varsigma}/\partial K > 0.$$

On the other hand, consumption at the end of high wage job search declines with turnover

costs: $\partial c_T/\partial K < 0$.

These results determine the changes in employment and search durations. Since the job search phase diagram for c_t and A_t is independent of turnover costs, an increased initial consumption c_0 along with decreased terminal consumption c_T implies that the new solution is on lower path but with larger first period consumption. Given higher initial consumption, it follows that assets rise at the outset of high wage search: $\partial A_D/\partial K > 0$.

On the lower trajectory, consumption for a given asset level falls:

$$\partial c(A_t)/\partial K < 0.$$

With more assets being consumed at a slower rate, the duration of search necessarily rises[15]: $\partial T/\partial K > 0$. As consumption during low wage employment rises, the rate of asset accumulation declines. However, at the termination of the low wage job at time D, the worker starts high wage job search with higher consumption - recall that c_0 has increased. From the phase diagram, the worker must arrive with higher assets, A_D. To accumulate a larger asset level with a slower accumulation rate requires that the duration of low wage employment increases: $\partial D/\partial K > 0$.

Increased turnover costs diminishes a worker's willingness to engage in search.[16] As

[15] Consumption as a function of the length or duration of search is in general ambiguous: $\partial c_t/\partial K \gtreqless 0$. For low levels of t this derivative is clearly positive as $\partial c_{t=0}/\partial K > 0$ but depending on risk aversion, the decline in consumption may be more rapid under higher turnover costs so that this derivative becomes negative.

[16] If turnover costs are zero, workers would work at bad jobs and then search for infinesimally short periods. For $K = 0$, a "chattering" solution between employment and search results.

search becomes more distant, low wage workers who are accumulating assets in order to eventually seek high wage employment become less willing to sacrifice today for more remote rewards. Low wage workers stay longer and consume more in low wage jobs. When they do switch to job search, they arrive prepared to search longer to offset the possibility of future turnover costs. They do so by arriving with higher assets and by consuming less given asset levels.

3.4 Wages

The individual's response to wages is less transparent, with results primarily available for initial and terminal consumption. It is straightforward to establish that a pay rise in bad jobs increases consumption during low wage employment. Likewise, consumption at the end of high wage job search rises with low wages:

$$\partial c_0/\partial w_L = \partial \bar{\varsigma}/\partial w_L > 0; \quad \partial c_T/\partial w_L > 0$$

(See the Appendix for details.) As low wage pay w_L does not affect the phase diagram, the new consumption path is a higher trajectory accompanied by a higher initial value. Consumption given assets rises $\partial c(A_t)/\partial w_L > 0$; however, although c_0 has risen it is not possible to graphically determine whether assets at $t = 0$ are higher. Given a small rise in initial consumption on the new path, the duration of search will fall. For a sufficiently large rise we get the opposite effect. Given this ambiguity, it is not possible to tell (at

this point) the effects on employment or search duration as well as asset accumulation.[17] Although increased consumption reflects higher income from bad jobs, it is unclear how individuals alter the allocation of time between work and search.

The individual's response to changes in the high wage is less revealing. As pay in good jobs improves, initial as well as terminal consumption both decrease:

$$\partial c_0/\partial w_H = \partial \bar{\varsigma}/\partial w_H < 0; \quad \partial c_T/\partial w_H < 0$$

With more attractive good jobs, low wage workers save at a higher rate in order to facilitate search. Now, however, the phase diagram shifts with w_H changes. For a given the terminal condition, there is higher consumption at each asset level - the consumption paths rotate upward. As such, little can be inferred regarding consumption while searching. Workers consume less toward the end of search activity (when assets are low) reflecting the greater return to search. Consumption at the outset of search is also lower although the way in which consumption given relatively high assets responds is undetermined. Likewise, the steeper path is balanced by a fall in initial and terminal consumption so working out the duration of search and the initial asset level cannot be done diagrammatically.

[17] Analytically, these effects depend on the solution of differential equation solution for c_t. Given the structure of this differential equation, the outcome is likely to depend on third derivatives for $u(c_t)$.

4 Conclusions.

If access to credit is limited (especially when young or unemployed) but "bad" jobs are easy to come by, then job seekers can use short term employment in such undesirable jobs as a way to finance consumption during subsequent unemployed search for "good" jobs. This paper provides both an empirical exploration and a theoretical exploration of this idea. The empirical analysis presented above adds to a small but growing literature that documents the importance of short-term and temporary employment to new entrants and displaced workers. It also draws out some aspects of labour market transitions that have received little attention. First, the data contradict the notion that quits into unemployment do not occur. Second, the consumption levels of temporary workers are starkly different from those back in "good" jobs and very similar to current searchers. Their savings behavior is quite different: savings rates among the less satisfied employed are as high or higher than those back in "good" jobs (and much higher than the unemployed).

Although somewhat at odds with conventional views of the labour market, these observations line up well with the model presented here. In an economy with credit constraints, readily available low paying jobs and more difficult to find high paying jobs, individuals may work in "bad" jobs to accumulate assets which subsequently fund search for "good" jobs. Workers in this economy, of course, prefer high wage employment but to secure one of these good jobs they must first engage in uncertain search. To finance consumption during this search, job seekers eat into assets - debt financing is not available. To some extent workers without assets can overcome the credit constraint by accepting low paying jobs. Such employment is readily available but hinders the ability to search for

high pay work. Low wage jobs therefore become temporary positions that fund subsequent job search. If the ensuing search is unsuccessful, workers repeat their asset accumulation in low wage employment. As a result, voluntary planned quits occur in a cyclical pattern that provides an explanation for a series of short job durations (at low wages) followed by employment at high wages.

Although the fundamental contribution of the model is to demonstrate, analytically, a mechanism for endogenous quits, the cyclical migration between sectors also provides insights into Harris-Todaro (1970) economies. While looking for good jobs, workers trade off the benefits of immediately available low wage work against those of unemployment. Here, however, there are explicit flows between sectors as workers move in and out of low wage employment. Wages and the costs of moving across sectors determine the size of these flows. Low turnover costs generate rapid movements between high wage job search and low wage employment.

Job turnover crucially relies on a financial market imperfection, the no-debt constraint. The credit constraint thus has new implications for the distribution of income. In the literature, it has been shown that borrowing constraints can affect the distribution of income by restricting human capital investment choices. Here, capital market imperfections have further implications for the distribution of income as they alter job flows. Without constraints, workers are of course better off ex ante although ex post some will be unlucky and be resigned to low wage employment with debts.

Like all models, ours involves a number of abstractions, two of which merit further discussion. First, we assume no on-the-job search. This is clearly extreme. All that

is required is that unemployed search be sufficiently more productive than on-the-job search. It is easy to imagine circumstances where unemployed search offers considerable advantages over on-the-job search. Among these is the possibility that an unemployed worker can travel to search in a distant labour markets. In this case, the turnover cost in our model (paid when search commences) has a natural interpretation as a travel cost as discussed in Section 3. It is interesting to note that in the data, respondents who quit (including those who quit for reasons other than to take another job) have relatively high rates of geographic mobility.

The second abstraction which merits comment is the assumption that good jobs last forever (that is, as in Danforth (1979), there is an absorbing state). Obviously, if one thinks about displaced workers, this is an assumption one would prefer to do without. This technical convenience, however, greatly simplifies the analysis and allows us to obtain analytical results. Models of search and consumption with exogenous job destruction can be studied numerically (see, for example, Algan et al., (2003), Lentz (2002), and Rendon (2002)), but obtaining analytical results has proved difficult.[18] Lentz and Tranaes (2003) present an interesting alternative: they introduce a wealth lottery to ensure concavity of the value function. To do without any such technical assumptions is currently beyond our abilities.

There are other abstractions as well. Labour market opportunities are certainly more varied than described, and we have not included realistic features of unemployment insurance systems in our analysis. Stripping down the set-up allows us to highlight the

[18] These papers do not demonstrate the cyclical savings mechanism derived in this paper.

critical roles played by wage differentials, credit constraints, and turnover costs, and to prove that cycles of search and employment can arise in such an economy. Including more features would obscure this focus but would most likely not undermine the central results of the paper. Such extensions are also required if we wish to bring the theory to the data in a satisfactory way.

These abstractions not withstanding, we note that the turnover generated by the model provides an explanation for the propensity of recent job losers to take temporary work - as reported by Farber (1997) and demonstrated here in Canadian data. These episodes of temporary work detract from time spent searching for a "good" job, and hence provide a partial explanation for the persistence of earnings losses after displacement that have been documented by Jacobson, Lalonde and Sullivan (1993) and others. The model generates the pattern of high savings and low consumption by workers in temporary jobs that we documented in the COEP data. Finally, the model also suggests a relationship between assets at job loss and the duration of initial search, as has been recently investigated in Stancanelli (1999), Bloemen and Stancanelli, (2001), Bloemen, (2002) and Algan et al. (2003).

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APPENDIX:

Proof of Claims 1-3

Claim 1: For any fixed $\bar{D}$ where $0 < \bar{D} < \infty$, define

$$V_{\bar{D}}(A_0;T) = \max_{\varsigma_t, c_t} \int_0^{\bar{D}} u(\varsigma_t)e^{-rt}dt + e^{-r\bar{D}} \left\{ \int_0^T u(c_t)e^{-(r+\alpha)t}dt + \frac{\alpha}{r}\int_0^T u(rA_s + w_H)e^{-(r+\alpha)s}ds + e^{-(r+\alpha)T}u(rA_0 + w_L)/r \right\}$$

subject to

$$(i) \qquad A_0 + \int_0^{\bar{D}} (w_L - \varsigma_t)e^{-rt}dt - e^{-r\bar{D}} \int_0^T c_t e^{-rt}dt - e^{-r(\bar{D}+T)}A_0 \geq 0$$

and

$$(ii) \qquad e^{-r(\bar{D}+T)}A_T \geq 0$$

for $K = 0$. This is the basic decision problem but with exogenous $D, T, A_T = A_0$ and permanent low wage employment after job search - $V(A_T) = u(rA_0 + w_L)/r$. As such the same first order conditions (1) and (2) continue to apply along with the budget constraint.

To establish that it is optimal to search when K is small, it is sufficient to establish that

$$\Delta \equiv V_{\bar{D}}(A_0;T) - u(rA_0 + w_L)/r > 0$$

for some $T > 0$. Evaluated at $T = 0$, $V_{\bar{D}}(A_0;T)_{|T=0} = u(rA_0 + w_L)/r$ and hence $\Delta = 0$.

Moreover, differentiation of $V_{\bar{D}}(A_0;T)$ and the budget constraint evaluated at the optimal choices for ς_t, c_t gives

$$\begin{aligned} d\Delta &= e^{-rD-(r+\alpha)T}\left[-u'(c_T)(c_T - rA_0) + u(c_T) + \frac{\alpha}{r}u(rA_0 + w_H)\right. \\ &\quad \left. -\frac{r+\alpha}{r}u(rA_0 + w_L)\right] dT \end{aligned}$$

As $T \to 0$, $c_T \to \bar{\varsigma} = rA_0 + w_L$. Therefore

$$\lim_{T\to 0}\frac{d\Delta}{dT} = e^{-rD}\left[\frac{\alpha}{r}\left(u(rA_0 + w_H) - u(rA_0 + w_L)\right) - u'(rA_0 + w_L)w_L\right] > 0$$

Therefore, $\Delta > 0$ for some $T > 0$ given the assumed condition. QED

Claim 2: Since $c_t > rA_t$, if search is finitely long then $\lim_{t\to\infty} A_T = 0$. Given $A_t = 0$, consumption equals zero during search: $c_t = 0$. For search not to be too attractive, it is sufficient to establish that at some point the worker must strictly prefer low wage employment over search with zero consumption. If the worker chooses to never move into low wage employment so that $T \to \infty$, the expected lifetime utility of having zero assets is given by

$$\frac{\alpha}{r}\int_0^T u(w_H)e^{-(r+\alpha)t}dt = \frac{\alpha}{r(r+\alpha)}u(w_H)$$

A worker with zero assets can always work indefinitely at a low wage job with payoff: $u(w_L)/r$. Given these options, the result follows. QED

Claim 3: To prove this claim, we establish four intermediate results. Recall that the assumptions in Claims 1 and 2 ensure that $0 < T < \infty$ and $D < \infty$.

Lemma 4 $V'(A_0) = u'(c_0)$

Proof: This is a familiar envelope result. Total differentiation of $V(A_0)$ gives

$$V'(A_0)dA_0 = \frac{1-e^{-rD}}{r}\left[u'(\bar{\varsigma}) - \alpha\int_0^T u'(rA_S + w_H)e^{-\alpha s}ds\right]d\bar{\varsigma}$$

$$+e^{-rD}\int_0^T\left\{u'(c_t)e^{-\alpha t} - \alpha\int_t^T u'(rA_s + w_H)e^{-\alpha s}ds\right\}dc_t e^{-rt}dt$$

$$+e^{-rD}\left\{u(\bar{\varsigma}) - r\int_0^T u(c_t)e^{-(r+\alpha)t}dt - \alpha\int_0^T u(rA_s + w_H)e^{-(r+\alpha)s}ds\right.$$

$$\left.-re^{(r+\alpha)T}V(A_T) + \alpha e^{rD}\left[rA_0 + w_L - \bar{\varsigma}\right]\int_0^T u'(rA_s + w_H)e^{-\alpha s}ds\right\}dD$$

$$+e^{-rD-(r+\alpha)T}\left[u(c_T) + \frac{\alpha}{r}u(rA_T + w_H) - (r+\alpha)V(A_T)\right]dT$$

$$+e^{-rD-(r+\alpha)T}V'(A_T)dA_T + \alpha\int_0^T u'(rA_s + w_H)e^{-\alpha s}dsdA_0$$

Plugging in the first order conditions gives

$$V'(A_0)dA_0 = \mu_1 \left\{ \frac{1-e^{-rD}}{r} d\bar{\varsigma} + e^{-rD} \int_0^T dc_t e^{-rt} dt + e^{-r(D+T)} \left[c_T - A_T \right] dT \right.$$

$$\left. -e^{-rD} \left[w_L - \bar{\varsigma} + r \int_0^T c_t e^{-rt} dt + r e^{-rT} A_T + rK \right] dD + e^{-r(D+T)} dA_T \right\}$$

$$+\mu_2 e^{-r(D+T)} dA_T + \alpha \int_0^T u'(rA_s + w_H) e^{-\alpha s} ds \, dA_0$$

Likewise, differentiation of the budget constraint gives

$$dA_0 = \frac{1-e^{-rD}}{r} d\bar{\varsigma} + e^{-rD} \int_0^T dc_t e^{-rt} dt + e^{-r(D+T)} \left[c_T - A_T \right] dT$$

$$-e^{-rD} \left[w_L - \bar{\varsigma} + r \int_0^T c_t e^{-rt} dt + r e^{-rT} A_T + rK \right] dD + e^{-r(D+T)} dA_T$$

Combining gives

$$\begin{aligned} V'(A_0)dA_0 \;\; = \;\; & \left\{ \mu_1 + \alpha \int_0^T u'(rA_s + w_H) e^{-\alpha s} ds \right\} dA_0 \\ & + \mu_2 e^{-r(D+T)} dA_T + e^{-rD-(r+\alpha)T} dV(A_T) \end{aligned}$$

From the Kuhn Tucker condition in (9), it is straightforward to demonstrate that $\mu_2 e^{-r(D+T)} dA_T = 0$ and hence

$$V'(A_0) = \mu_1 + \alpha \int_0^T u'(rA_S + w_H) e^{-\alpha s} ds = u'(c_0)$$

from equation (1). QED

Lemma 5 *If* $A_T > 0$, *then* $V''(A_T) < 0$

Proof: $A_T > 0$ implies that $\mu_2 = 0$ and therefore from the first order conditions

$$e^{-\alpha T}V'(A_T) = \mu_1 = e^{-\alpha T}u'(c_T).$$

Differentiation gives

$$V''(A_T) = u''(c_T)\frac{dc_T}{dA_T}.$$

Likewise differentiation of the first order condition

$$(r+\alpha)V(A_T) = u(c_T) + \frac{\alpha}{r}u(rA_T + w_H) + u'(c_T)(rA_T - c_T)$$

gives

$$(r+\alpha)V'(A_T) = \alpha u'(rA_T + w_H) + ru'(c_T) + u''(c_T)(rA_T - c_T)\frac{dc_T}{dA_T}.$$

Combining yields the desired result:

$$V''(A_T) = \frac{\alpha\left[u'(c_T) - u'(rA_T + w_H)\right]}{rA_T - c_T} < 0.$$

QED

Lemma 6 *If* $D > 0$, *then* $V''(A_0) = 0$

Proof: If $D > 0$, then $\bar{\varsigma}$ is well defined and satisfies

$$rV(A_0) = u(\bar{\varsigma}) + u'(\bar{\varsigma})(rA_0 + w_L - \bar{\varsigma}).$$

Therefore,

$$rV'(A_0) = ru'(\bar{\varsigma}) + u''(\bar{\varsigma})(rA_0 + w_L - \bar{\varsigma})\frac{d\bar{\varsigma}}{dA_0}$$

From Lemma 4 and the result that $\bar{\varsigma} = c_0$, it follows that $d\bar{\varsigma}/dA_0 = 0$. Further, Lemma 4 yields

$$V''(A_0) = u''(\bar{\varsigma})\frac{d\bar{\varsigma}}{dA_0} = 0$$

QED

Lemma 7 *Given* $A_0 > 0,\ A_T < A_0$.

Proof:

Case 1: $D = 0$. As $0 < T < \infty$ and $c_t > rA_t,\ \ A_T < A_0$ follows immediately.

Case 2: $D > 0$. Suppose $A_T \geq A_0$. Define assets at the end of low wage employment by

$$A_D = e^{rD}A_0 + \frac{e^{rD} - 1}{r}(w_L - \bar{\varsigma}).$$

Notice that $A_D > A_T$ by the logic used in Case 1. Individuals with assets $A \in [A_0, A_D)$ choose positive low wage employment, $D > 0$, and hence from Lemma 6, $V''(A) = 0$. As a result

$$V'(A) = V'(\tilde{A})\ \forall\ A, \tilde{A} \in [A_0, A_D).$$

Since $0 < T < \infty$, $c_0 > c_t$. Moreover, $A_T > 0$ implies $\mu_2 = 0$ so that $V'(A_T) = u'(c_T)$. From Lemma 1, we therefore get the contradiction that

$$V'(A_T) = u'(c_T) > u'(c_0) = V'(A_0).$$

QED

These lemmas are now used to establish the claim. Suppose $A_T > 0$. By Lemma 5, $V''(A_T) < 0$. By Lemma 7, $A_T < A_0$. From Lemma 6 and Claim 1, $V''(A_T) = 0$, a contradiction. QED

Comparative Static Calculations

For the analysis here let $A_0 = A_T = 0$. These asset levels although optimal can be treated as exogenous in which case equations (7) and (9) become unused. At this point it is useful to define V, the value of starting (and completing) a cycle of low wage work followed by search for high wage employment where initial and terminal assets equal zero ($A_0 = 0$ and $A_T = 0$) :

$$V = \frac{1 - e^{-rD}}{r} u(\bar{\varsigma}) + \quad e^{-rD} \left\{ \int_0^T u(c_t) e^{-(r+\alpha)t} dt \right. \tag{10}$$
$$\left. + \frac{\alpha}{r} \int_0^T u(rA_s + w_H) e^{-(r+\alpha)s} ds + e^{-(r+\alpha)T} V \right\}$$

where the choice variables, $\bar{\varsigma}, c_t, T$ and D equal their optimal values as defined by the first order conditions (1), (2), (5), (6) and (8).

Rearranging the equations produces separate equations for $\bar{\varsigma}$ and c_T as functions of V and exogenous parameters

$$rV - u(\bar{\varsigma}) - u'(\bar{\varsigma})\left[w_L - \bar{\varsigma}\right] = 0 \tag{11}$$

$$u(c_T) + \frac{\alpha}{r}u(w_H) - (r+\alpha)V - u'(c_T)c_T = 0 \tag{12}$$

As shown in Lemma 4 (as part of the proof to Claim 3),

$$\mu_1 + \alpha \int_0^T u'(rA_S + w_H + w_F)e^{-\alpha s}ds = u'(\bar{\varsigma})$$

Differentiation reveals that after some manipulation:

$$\begin{aligned}[1 - e^{-rD-(r+\alpha)T}]dV &= -e^{-rD}u'(\bar{\varsigma}) \cdot dK + \frac{1 - e^{-rD}}{r}u'(\bar{\varsigma}) \cdot dw_L \\ &\quad + e^{-rD}\frac{\alpha}{r}\int_0^T u'(rA_s + w_H)e^{-(r+\alpha)s}ds \cdot dw_H\end{aligned}$$

From (11),

$$rdV - u''(\bar{\varsigma})\left[w_L - \bar{\varsigma}\right] \cdot d\bar{\varsigma} - u'(\bar{\varsigma}) \cdot dw_L = 0$$

Plugging in for rdV gives

$$d\bar{\varsigma} = \frac{1}{[1-e^{-rD-(r+\alpha)T}]u''(\bar{\varsigma})\,[w_L-\bar{\varsigma}]}\left\{-re^{-rD}u'(\bar{\varsigma})\cdot dK\right.$$

$$\left.-e^{-rD}(1-e^{-(r+\alpha)T})u'(\bar{\varsigma})\cdot dw_L+e^{-rD}\alpha\int_0^T u'(rA_s+w_H)e^{-(r+\alpha)s}ds\cdot dw_H\right\}$$

Likewise, from (12),

$$\frac{\alpha}{r}u'(w_H)\cdot dw_H-(r+\alpha)\cdot dV-u''(c_T)c_T\cdot dc_T=0.$$

Again plugging in for dV gives

$$dc_T=\frac{-(r+\alpha)}{r[1-e^{-rD-(r+\alpha)T}]u''(c_T)c_T}\left\{re^{-rD}u'(\bar{\varsigma})\cdot dK\right.$$

$$-(1-e^{-rD})u'(\bar{\varsigma})\cdot dw_L+\alpha\left[\frac{(1-e^{-rD})}{r+\alpha}u'(w_H)+\right.$$

$$\left.\left.+e^{-rD}\int_0^T\left[u'(w_H)-u'(rA_s+w_H)\right]e^{-(r+\alpha)s}ds\right]\cdot dw_H\right\}$$

Figures and Tables

Table 1: Separation Reasons and Unemployment

	initial jobless spell, weeks	search	search hours / week	search expenditures $ / week
	median	%	mean*	mean*
Layoff due to:				
business conditions	7	72%	15	34
temporary job / contract ended	9	69%	13	32
job was seasonal	12	66%	12	36
other	5	63%	14	31
Quit				
to take another job	<1	16%	12	23
other	6	58%	15	32
Dismissed or Fired	7	84%	17	31
Illness or Injury	10	31%	17	52
Other	3	52%	14	39
Not stated	3	76%	16	20

* mean conditional on searching

Table 1: Separation Reasons and Unemployment, Continued

	geographic move	UI receipt	# obs.	% of Sample
Layoff due to:				
business conditions	6%	73%	1,847	25
temporary job / contract ended	7%	73%	1,679	23
job was seasonal	6%	71%	859	12
other	5%	70%	390	5
Quit				
to take another job	14%	18%	665	9
other	19%	20%	541	7
Dismissed or Fired	8%	51%	207	3
Illness or Injury	5%	61%	386	5
Other	9%	51%	754	10
Not stated	0	28%	25	<1

Table 2: First Interview Employment Status (3rd Quarter after Job Loss)		
1. First Spell of Unemployment (UE-1)	33.3%	
2. Subsequent Spell of Unemployed (UE-2)	18.2%	
3. First Spell of Post-displacement Employment	37.6%	
4. Subsequent Spell of Post-displacement Employment	10.9%	
Of 3 :		
(A) Not Expecting Job to Last 1 Year	11.0%	(29.3%)
(B) Expecting Job to Last 1 Year	26.6%	(70.8%)
(i) Current Job Worse Than Job Displaced From	9.4%	(24.9%)
(ii) The Same or More Satisfied with Current Job	28.2%	(75.1%)
	———	
Total Temporary Work: 2.+4.+3.(A)	40.1%	
Total Temporary Work: 2.+4.+3.(i)	38.5%	

Table 3: Pair-wise Rank- and t- tests of Common Expenditure Change Distribution

	UE-2	E-1	E-2
	subsequent spell	less satisfied	same or more satisfied
UE-1	1.94 (0.05) 1.773 (0.183)	0.881 (0.38) 0.889 (0.343)	5.03 (< 0.001) 22.98 (< 0.001)
UE-2	-	0.764 (0.45) 0.058 (0.809)	2.26 (0.025) 7.20 (0.007)
E-1	-	-	2.90 (0.004) 5.27 (0.02)

Notes. The boxes give difference in means: t-stat p-value
KW rank test: $\chi^2(1)$ p-value

Table 4: Pair-wise Rank- and t-tests of Common Income Change Distribution

	UE-2	E-1	E-2
	subsequent spell	less satisfied	same or more satisfied
UE-1	1.14 (0.26) 2.207 (0.14)	2.77 (0.006) 6.91 (0.009)	9.29 (< 0.001) 79.3 (< 0.001)
UE-2	-	1.45 (0.149) 1.13 (0.287)	6.36 (< 0.001) 32.9 (< 0.001)
E-1	-	-	4.25 (< 0.001) 22.94 (< 0.001)

Table 5: Pair-wise Rank- and t-tests of Common Savings Rate Distribution			
	UE-2	E-1	E-2
	subsequent spell	less satisfied	same or more satisfied
UE-1	0.068 (0.95) 0.040 (0.842)	2.43 (0.016) 7.74 (0.054)	1.31 (0.19) 23.21 (< 0.001)
UE-2	-	2.17 (0.031) 4.990 (0.026)	1.00 (0.32) 12.68 (< 0.001)
E-1	-	-	0.46 (0.64) 0.507 (0.476)

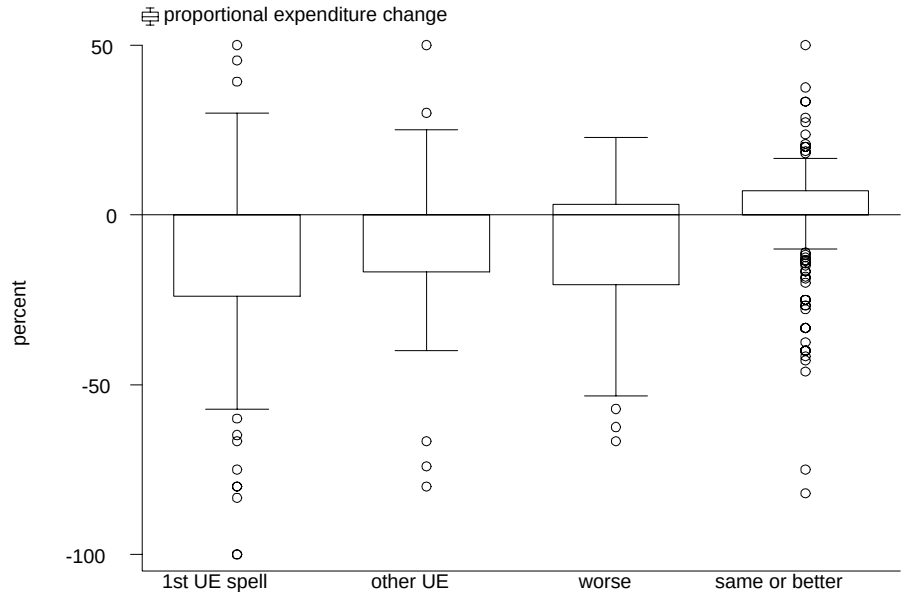

Figure 1: Proportional Expenditure Changes

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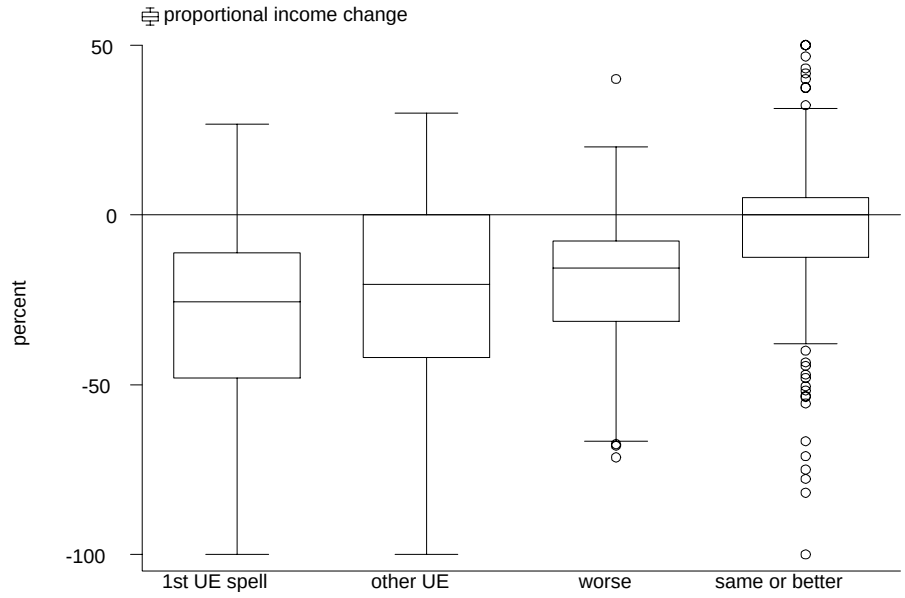

Figure 2: Proportional Income Changes

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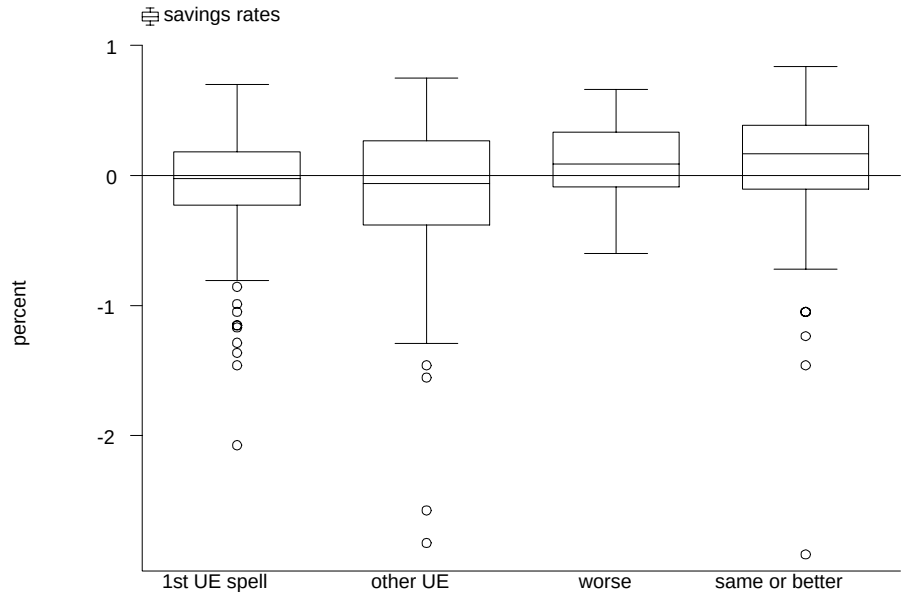

Figure 3: Savings Rates

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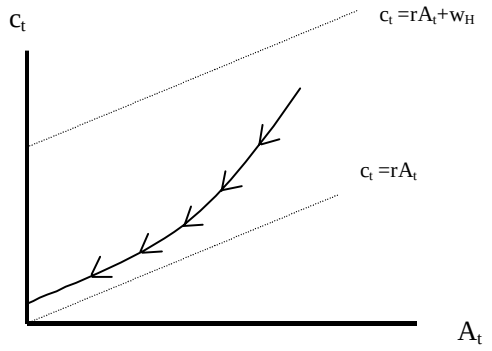

Figure 4: Consumption During Search

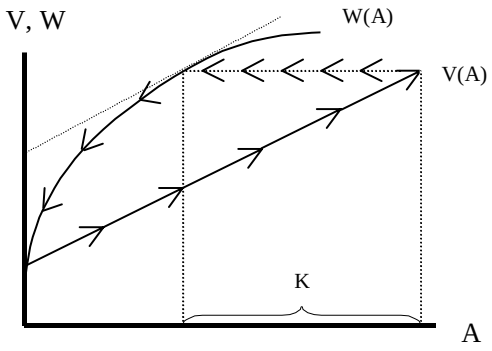

Figure 5: The Cyclical Pattern of Quits

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